Vehicle Performance – AE3330

Review: Two-parameter drag polar:

$Dc = Dc0 + kCl^2$

Two parameters you need to find: Cdo -> entire drag of aircraft.

K => coefficient

Good start for the early phases of design.

Is Cd dependent on alpha? Yes, because Cl is a function of alpha. (AoA)

BUT you can’t evaluate for all Cl. You need to be aware of stall.

More AERODYNAMICS for now

(page 2 has a drag polar graph with annotations)

Design craft for when Cdmin is NOT at Cl = 0, when Cdmin is above Cl=0, and this is called Clmindrag.

Three-parameter drag polar.

This is:

$Cd = Cdmin + k(Cl – Clmindrag)^2$

This may give you a better curve fit.

(there is a graph on page 2)

Drag polar

-Cd, Cl depend on Mach and reynold’s number

-If dependency on Mach is weak (M<.3)

And if dependency on Re is weak, which usually is the case when you're not close to Clmax)

--Then we canassume that the drag polar is constant with respect to M and altitude.

Meaning: Cd,0 and k are constant.

-If dependency on M is STRONG (transonic or supersonic) then the drag polar is only valid at specific M and alititudes.

Also true if dependency on Re is strong)

For high subsonic, it depends on how accurate you need to be 0.7 > M > 0.3

-DRAG POLARS are configuration-dependent, so they change with landing gear, flaps/slats, etc.

(A few example problems are on page 4 and 5

And 6

Propulsion

Intro

-Propulsion systems

-How is T generated by air-breathing engines

-What parameters affect T

-What is propulsive efficiency

-How are T and efficiencty related?

-How to use this info to select propulsive systems

-Idealized propulsive system

(picture on page 7)

A streamtube – no flow through the boundary of the stream tube

Inflow boundary where v_inf goes in

Outflow boundary where vj goes out

-represents a propeller or turbojet

Mass inflow = mass outflow

Rho-inf*v-inf*Ainlet = rho-j*v-j*A-outflow

Key assumptions: v_inf is constant across A_inflow and v-j is constanct across A_out

Fluid is inviscid and incompressible

Idealized prop key assumptions continued:

-no rotation in either inflow or outflow

-P = P-inf at inflow/outflow boundaries

-mass in = mass out

-no mass souirces/sinks inside control vol

-pressure forces on steam tube are negligible.

From physics: F = dp/dt

Force is the time rate of change of momentum (in this case, fluid momentum.

Consider delta-t, during which some amount of fluid with mass m goes through inflow boundary.

Momentum of this fluid: m*v_inf.

(derivation)

Yadda yadda

Equation for thrust:

T = m-dot(vj – v-inf).

Observations:

-T depends on vj-v-inf

-if vj increases, thrust increases

If v-inf equals vj, then thrust is 0..

-T depemds on m-dot

If m-dot increases, T increases.

That means if you increases the diameter of the propulsive system, T increases.

PROPULSIVE EFFICIENCY: eta_p = (useful power available) / (total power segmented)

Power = force * ditstance / time

OR

Force * velocity

Useful power available" P_A = TV-inf

Total power -> Pa + power wasted

P_t = Pa + power wasted.

Power wasted?

-wasted kinetimatic energy per unit mass left behind.

½*m(vj-v-inf)^2

-During tome interval delta-t, mass m of fluid is processed.

Power wasted: ½*m-dot(vj-v-inf)^2

Total Power:

P_total = T* V-inf + ½*m-dot(vj-v-inf)^2

ANYWAY

We get

Eta_p = 2 / (1 + vj/v-inf))

This is ideal propulsive efficiency

A system with specific vj, v-inf values

This and the thrust equation are the fundamental starting point for propulsion.

WE HAVE A TRADEOFF/ between thrust and efficiency.

Highest efficiency is when vj=v-inf. But that's when T = 0.

Propellers

-vj not much greater than v-inf

-process a large amou t of fluid per unit time

-at low speeds M<<.4

Propellers offer good tradeoff between T and eta_p.

Drawbacks:

-limited by tip M#

-both T and speed

b/c of this, props are not suitable for any type of high speed aircraft.

Turbofans and Turbojets

-turbojets have high value of vj/v-inf, which means that the efficiency is lowe.

-for a given T at v-inf, turbojet processes less fluid per unit time than a propeller, but imparts a higher vj

-turbojets -> not limited by M#'s

-good options for transonic and supersonic flight

-turbofans -> hybrid between turbojet and propellers

Time for some talk on reciprocating engines!

-most aircraft use 4-stroke Otto cycle

-good combination of T, eta-p. and WEIGHT.

For propellers, it's better to talk about Power rather than T

Equation for

P_sbd, "Shaft brake pwer) is on page 12

Psbd = eta_mech * RPM * d * P_e / 120

N_cech is mechanical efficiency

RPM – revolutions per minute

D – engine displacement – (piston area * piston stroke * # of pistons

P_e = mean effective pressure

Reciprocating Engines

(a picture of a piston is on page 13)

BMEP -> brake mean effective pressure

BMEP = eta_mech * P_e

(these are just the things we can easily measure)

Again – reciprocating engines used with propers have this equation:

Psbd = n_mech * P_e * RPM * d / 120

Fuel consumption

C = specific fuel consumption

It has consistent units

It is the weight of fuel burned / (unit power) (unit time)

So it endes up being m^-1

Or ft^-1

SPC – specific fuel consumption is in inconsistent units.

Sometimes in hr^-1, sometimes hp (horsepower)

Engine performance

-Assume Psbd constant w/r/t V_inf

SFC constant w/r/t/ v_inf and altitude

If engine is normally aspirated (taking in air normally) as altitude uncreases, power decreases (lapse)

Since rho decreases, therefore P_e decreases

At a constant T, Psbd = Psbd0 (rho/rho_0)

Supercharging

-Increasing Power @ sea level and delaying lapse in engine power w/ altitiude

-ambient air is compressed so that rho and P are increased before being provided to the engine

→ supercharger or turbocharger

(superchargers are where the compressor is driven mechanically)

Turbochargers are where the compressor driven by turbine in engine exhaust

A graph of Psbd for propellers is on page 15

Propeller performance

-prop is a rotating wing in which L is used for T

V_inf is freestream vel

R = radiusat this station

W == rotation velocity

Beta = pitch (geometric)

Alpha_g = geometric AoA

L* = lift/unit radiusparallel to relative wind

T* = Thrust/unit radius

T* = L*cos(beta – alpha_g)

(pictures and equation on page 15)

For a given V-inf, as r changes, beta has to change to keep propeller airfoil at optimum alpha

Alpha_g = alpha_g,opt

Prop blades are twisted in order to maintain alpha_g opt

(look at equation for beta on page 16)

And picture on page 16

Propeller performance….

V_inf / rw -> key performance parameter

Similarly,

J = V_inf / ND

J = advance ratio

N = revolutions / sec

D = diameter

OR

J = pi*V_inf / r_tip*w

(w is angular vel, and r_tip is the prop radius at tip)

EFFICIENCY AND J

PA = n_pr * Psbp

PA = power available to generate T

N_pr = prop efficiency (between 0 and 1)

Psbp = Power produced.

The goal is to maximise n_pr.

N_pr is a function of J.

For a fixed pitch propeller, there is only one optimal eta_pr.

For a variable pitch propeller., you can extend n_pr to more velocities.

Variable pitch props allow for constant speed prop.

Eta, engine performance is optimal at specific N. Constant speed is best engine performance. Variable pitch is the best prop eta.

SUMMARY OF PROPELLER PERFORMANCE

PA = n_pr * Psbp

J = v_inf / ND

N_pr = f(J)

For constant speed prop (w*r)

Psbp = constant b/ v_inf

Psbp = Pspb0*(rho/rho0) normally aspirated

SFC is constant with altitude and V-inf/

Turbojets

-type of jet (reaction) engine in which all of the air flowing into the engine is processs through the combustion chamber

-thrust is produced by accelerating the air

(picture of turbojet on page 18)

-the reaction is directly providing the thrust.

TURBOJECT THRUST EQUATION

T = m-dot(vj-v-inf)/

You can assume that a lot of things are negligible, so it's just the regular ol' thrust equation.

Turbojet

--At subsonic speeds

Vj is assumed constant, therefore thrust is mostly insensitive to velocity, M#

Thrust can be said to be constant with M number.

At supersonics speeds, Thrust = T(m-inf=1) * (1 + 1.18(M-inf – 1))

(eq on page 20.

But this doesn't mean you can go faster forever

As altitude increases, T = T0(rho/rho0)

Fuel consumption:

Turbojet

Weight / thrust / time

C_t -> thrust-specific fuel consumption

C_t = N/N*s = ½

Cl = 1/s

TSFS is the inconsistent unit Thrust specific fuel consumption: 1/hr

---

For M-inf < 1, TSFC = TSFCMinf = 0 + kM_inf

M-inf > 1, TSFC = constant w/ M_inf

Altitude: TSFC is constant both sub and supersoncic

Turbojet summary of equations: is on page 21!!

Advantages and disadvantages:

-wide variety of sizes

-T/W is about 4-8, thrust to weight ratio

-light and smaller than comparable reciprocating engine with prop

-well-suited for high alt + speed gflight M-inf > 0.8

Can be used at lower altitudes, but other options are better in terms of T and efficiency.

Turbojet TFSC is higher than other devices

Also there are afterburners – another combustion chamber at the end of the engine. Super inefficient but generates a ton of thrust.

Turbofans

-a type of jet (reaction) engine in which some (but not all) the air flowing into the engine is processed through the combustion chamber

-and the rest of the air goes into cor (turbojet)

-other air bypasses the combustion chamber and is accelerated by a ducted fan

Thrust is produced by accelerating air bother by core and fan

-fan is driven by a turbine

(picture on page 22 and 23

30% to 60% of thrust is gerneated by the fan

Categories of Turbofans:

Low BPR 0<BPR < 1

High BPR 4 < BPR < 6

Thrust (you can make similar assumptions that you made with turbojet

T = (mc-dot + mb-dot (v-inf – v_j)

Trying to maximize BPR.

At subsonic speeds for high BPR

T = T_vinf=0 * A * M ^-n

A, n, and T_vinf = 0 are engine-specific constants.

Fuel consumption

C_t, TSFC (1/2, 1/hr)

Subsonic, high BPR” $c_t = B(1 + kM_{inf})$

B and k are engine constants

C_t = constant w/r/t altitude

$M < 1$

Summary of Turbofan equatons is on page 24!

Turbofan advantages and disadvantages

-wide range of sizes

0high efficiency at Mach# of transonic speeds

-can work up to about M = 1.5 but turbojets are better at higher mach numbers

-T/W is about 5 or 6

-simpler mechanicall than turboprops and can operate at higher Mach #’s

-less noise than equiv. turbojet or turboprop

-mixing hot and cold air for some reason leads to noise reduction

---

LASTLY

Turboprops!

-A type of reaction (jet) engine in which most of the energy in the jet flow is converted to mech power to drive a propeller

-Most of the thrust T is produced by the propeller

-5% - 15% powered by jet exhaustsed

(pictre on page 27)

Talk about Power b/c it’s a propeller (easier to talk about because it is constant)

Available Power

PA = (Tp + Tj) *v_inf = Tp*V_inf + Tj*v_inf

PA = n_pr * Ps + Tj*V-inf

Sometimes writing as something else

(equations written better with more explanation on page 27)

Power (thrust)

PA constant with M_inf (though limited to operating range)

PA = PA0(rho/rho0)^n

N = 0.7 – no change for bypass

PA,0 = PA at sea leveloperating range = 0.2 < M < 0.7

Fuel consumption

-defined w/r/t T sometimes

-defined wrt P _. Need to specift PA, Ps, Pes

w-dot fuel (rate at which fuel is being burned)

(there is a description of all the Cts, Ca, Cs, and Ces are on page 28)

Advantages and disadvantages of turboprop

-somewhat limited thrust range: 375-7500 kW

-high efficienct 0.4 < M < 0.7

-power to mass ratio 3.3 kw/kg

-turbo prop weight is about 33 percent of equipvalent reciprocating engine and propeller

-higher SFC than recip. Engine and prop, lower than turbojet

-turbofan burns less at higher alt

-turboprop is more expensive initially but less maintenance and more reliable than a recip. Engine and prop

-turboprop is heavier than equiv turbojet or fan b/c you need a lot of geat systems – you get more flexibility with turbofan

ALTERNATE ENGINE TYPES

-turboshaft

-intended to produce shaft power only

-used for rotocrafts, APU's

A couple of example problems are on page 30

DONE with PROPULSION and ONTO Design

Aircraft Design

-Design Phases: preconceptual, conceptual, preliminary, detailed!

-proe-conceptual

-study mission requirements and marked

-conceptal

-analyse req.

-initial configuration

-preliminary

-CAD

--performance estimates

-subsystem design

-detail

-perats manufacturing

BUT AFTER THIS

-production, operation, support/upgrading retirement

(graph of design process is on page 31)

The goal is to move, shift plots

To increase knowledge earlier on, it's important to make your big assumptions with simple analyses.

WEIGHT ANALYSIS!

Initial weight analysis/estimate

W = Wc + Wp + Wf + We

Crew weight, payload weight, fuel weight, and empty

W0 = max gross takeoff weight

Analysis and derivation on pages 32 and 33

And 34

BACK TO DESIGN

SIZING -> weight and geometric scale of aircraft necessary to performa all segments

- ➔ Synthestsis - > coombnining all the disciplines together into one analysis
- ➔ -weight/geom (sizing
- ➔ -aerodynamics
- ➔ -propulsions
- ➔ -mission specific
- ➔ -structures
- ➔ -controls
- ➔ -manufacturing
- ➔ -etc.
- ➔ You can do an N^2 diagrma
- ➔ Goal in the end is to do synthesis and get consistent, mission-specific class level of fidelity optional, but need to add fidelity consistently throughout disciplines

---

NOW TIME FOR STEADY FLIGHT

Break this up into steady flight and accelerated flight.

Steady level flight T=D and L=W

Steady, level flight

T_R = thrust required

(There is a picture on page 36

Governing equations are on page 36

The thrust required equation is on page 37!

When is T_R minimized?T_R is minimized when L/D is maximized

For a TURBOJET

TDerivation of this is on page 37

If you fly slower, D increases, so velocity decreases, so Drag in creases (graph pof this is on page 37

Wge is T_R min???

T_R_min = W/(L/Dmax)

This is the same thing as W/ (Cl/Cdmax)

How to find that is….take the two-lparameter drag polar, mmanipulate it so you are getting Cl/Cd, then take the derivative w/r/t Cl and set it equal to 0. You end up with Cl = sqrt(Cd0 / k . That's when you get Cl/Cd) max

This is then when you get TR_min.

An equation for V-inf at L/Dmax follows on page 38.

The analysis is that as Cd0 increases, V-inf at L/D max decreases.

As k and W/S and altitude increases, v_inf at L/D max increase.

EQUATIONS THAT WERE MISSING from earlier notes are on pages 40, 41!!!!

Steady flight notes continued

REVIEW so far

TR_min -> L/Dmax

Cd = Cd0 + kCl^2

Cl = sqrt(Cdo/k)

L = W = Cl*q_inf*s

V_inf * sqrt(etc etc at L/D max

CONTINUING

Cl/Cd_max at TR_min -----

(these derivations are on page 45, and are j

Equations for TR as a function of velocity are on page 46 as well as V_inf as a function of TR.

Also a graph on page 46.

Summary of equations is on page 47.

POQW

POWER REQUIRED

As opposed to TR

(left off at page 48)

Pr - same set of assumptions:

Steady, level floght, ignoring epsilon

T = D, L = W

P = TV_inf

It looks similar, but it is different

The fuel constant tends to be constant w/r/t Power for propellers

P_r = T_r*V_inf = V_inf (q_inf*s*C_d0 + q_inf*skCl^2)

Cl = W/q_infs

P_R = /2*rho_inf * V-inf*s*Cd0 + k*2*W^2/rho_inf*v_inf*s

This is the drag due tp lift term (the last one

If you add the two drag terms together it's the P_r vs. V_inf graph, which looks like kinfd of a lopsided parabola.

V_iThe Power required equation is on page 48

Power required, continued:

(Page 49)

P_r = T_r*V_inf = D*V_inf = D/W*W*V_inf = W/(L/D * V_inf

V_inf = sqrt(2*W / (rho_inf) * s * Cl))

P_R = W/(L/D) * swrt(2W/ rho_inf*s*Cl) = (sqrt (2/rho*s) * W^(3/2) * Cd / CL^3/2

This is more cleanly represented on page 49

As weight increases, P_R increases

As rho_inf increases, ppower required decreases

As Power Power required for props is a function of CL^3/2 over Cd

As C: ^ 3/2 / Cd increases, Power required decreases.

Sp, PR_min corresponds to maximum CL3/2 / Cd/

Pr_min = sqrt(2/rho*s * W^3/2 / (Cl^32/Cd) (max

To find Cl and V_inf at Pr_min…..

What you do is take Cl^3/2 and place it over the two-parameter drag equation. Then you differentiate w/r/t Cl. Then plug it back into Cl^3/2 / Cd.

Some algebra later and you have that the Cl that corresponds to Pr min at prop = Cl = sqrt (3* CD0 / k)

This is for minimum power

V_inf = sqrt (2*W / rho_inf * s * Cl)

Plug Cl into that and you get V_infinity at Pr_min Minimum power required (page 50)

Which is sqrt(2W/rho_inf * s *sqrt(k/s*Dc0))

As Cd0 increases, V-inf at Prmin decreases

As k increases, v_inf at pr_min increases

As W/S increases, V_inf incresase

As altitude increases, rho_inf decreases, so v_inf at pr_min increases.

V_inf at Cl^3/2/Cd max = 4*sqrt 1/3) * v_inf (Cl/Cd) max

Therefore flying at minimum power is about ¾ the velocity of flying at minimum Thurst.

Find Cl^3/2 / Cd) max at Pr_min….

This derivation has been done before….the result is on page 50

Maxmum Airspeed!

Let's start with turbojets…

Subsonic, steady level flight

Cd = Cd0 + kCl^2

TA is constant regardless of Mach #

T = D

We have our velocity equiation for Thrust required

(it's on page 51)

So…to get V_max, you replace Tr with TA and take the positive of the square root options

Can you fly faster than max speed?

Yes, but not at level flight

Altitude affects V_max

TA at H2 is less than ` so Vmax2 might be less than VmaxH1.

(some graphs on pgage 52)

TA = TA@sealevel(rho/rho0) for turbojets

In general: problem approach:

- ➔ Graphically, TR + TA vs. V_inf
- ➔ → analytically
- ➔ Set TR = TA, solve for V_inf
- ➔ → this gets complex for some models!!!

Let's do this for a constant power device, as opposed to a constant Thrust device

(prop or turboprop)

Assumptions: drag polar valid

Psbp is constant

Variable pitch prop n_pr is constant

T+D / T is parallel to V_inf

Find v_max

We got P_req = ½ rho_inf * v_inf^3 * s * Cd + 2kW^2 / (rho_inf * V-inf * s)

P_r = Pa

Pa = ½ rho_inf * V_max^3 * s * CD + 2kW^2 / rho_inf * v_inf * s

We can solve this numerically or graphically

For props

PA = Pa0 (rho/rho0) ^n ; n = 0/7 for tprops

You can solve

Remember props don't really work for transonic speeds and supersonic speeds

---

Max speed is not really a constraining performance metric!

Other things like aeroelastic effects

Can make things go slower and structural

Stall Speed

Much for important

How slow can you go?

V_inf = sqrt(2W / rho_inf * s * Cl)

As W decreases, V_inf decreases

Alt decresases, rho increasaese and V_inf decreases

As s increrasesm v_inf increases

How can we reduce stall speed?

We raise C_l max! We can increases CL_max to resduce stall speed

Ooooh! Therefore we use high lift devices when landing

Now let's talk about Rate of Clomb, angle of Climb

Equations of motion for ROC (not accelerated climb) are on Page 56

ROC = excess power / Weight

(see graph on page 56

Solve these problems in the Power Space

Assume constant PA

For constant power device….

ROC_max is excess power max is Preq min

Rate of Climb

ROC = V_inf (T/W – D/W) it'

S approximately equal to V_inf (T/W – 1/ L/D)

As T/W oir L/D increases, ROC increases

If we assume the two-param drag equation

….follows is a derivation for ROC

It ends on Page 57.

It’s a new day

Let’s talk about Max theta (andlge of climb) for TURBOJETS

-ROC refers to climb/time

-Angle of climb refers to climb/distance

You can fly so you can reach either max angle of climb opr ROC…not both

(derivation on page 59)

Equation for V_thetamax on page 60

You can do the same calculation for turboprop

MaxROC for propeller

PA = constant (treated as such)

Excess power is maximized at PR_min

Which is CL^3/2 / Cd max

Therefore Cl = 3Cd0 / k (sqr root of that) at CL3/2 / Cd max

So VROCMAX is equal to sqrt(2/rho_inf) * k/3CD0)^1/2 * W/S)

We can assume small angles because the plane isn’t traveling at a great angle

ROC = V_inf * sintheta

We can assume small angles because we’re still in the design phase.

We can make a lot of assumptions because we’re doing “Back of the envelope calculations

So, the simplified version for ROC_max is on page 61.

IN GENERAL

ROC max decesarses as altitiude increases (not always)

-Power required increases and power available decresases as altitude increases (again, not always, but often)

MAX ROC _> Min PR for prop.

But for jet, not always the case.

This concludes discussion on ROC and angle – Remember when and why to use them

---

Ceilings

--Absolute ceiling

-The highest altitude an aircraft can fly

→ ROC_max = 0

-A few ways to find this

Solutions:

->graph OC_Max vs. altitude, interpolateor set ROC_max = 0 and solve for rho_inf

→Make sure to use the appropriate propulsive model

-THOUGH

Absolute ceilings are not practical

SO…

We have another ceiling called the

SERVICE CEILING

-This is the highest altitude at which it is practical to fly

-usually defined as altitude at which ROC_Max = 100 ft/min (which is 0.5 m/s)

There is a graph of H_abs and ROC Max on page 62

Time to climb

-time needed to reach specified altitude

-often measured at Sea Level

→ b/c we are measuring for the sake of comparison in design process. There's not a lot of realism here.

Tim to climb is the integral from 0 to h dh/ROC

So the minimum time to climb is the integral from 0 to h of dh/ROC_{max}

(formula and derivation on page 63

Solving this analytically sucks, so you can soklve this graphically, or by doing numerical integration

-assume analytical form of ROC_{max} that you can actually integrate.

Anyway, it looks like (graph on page 63)

Where t_{min} is the area under the curve of altitude vs. $1/ROC_{max}$

RANGE

Remember the weight discussion?

-How far can we fly?

Range is mostly a function of weight.

To help with discussion, we define it for a few weights:

W_0 is the weight of aircraft at the start of cruise

(cruise is were we care about range)

We can't to T/O and landing at max range conditions anyway.

W_f = weight of fuel available at any specific point (for remainder of cruise)

(does not count what you need for margin of landing)

W_1 = weight of airfract at the end of the cruise

W = weight of A/C at any point

$W = W_1 + W_f$

(picture of cruise and weight at the top of page 65

We haven't made any assumptions by this point about constant altitude of velocity

$dW/dt = d/dt(w_1 + w_f) = dW_f/dt$

(THE WEIGHT EQUATION, on page 65

Assuming $dW_f/dt < 0$. We are losing weight as we go

Let's start at turbojets

C_t = thrust specific fuel consumption

=W of fuel burned / T * deltat

dWf/dt = -weight of fuel burned / delta-t

We can relate the two…..

C_t = -1/T * dWf/dt or dt = -1/ctT * dWf

Here, thrust is thrust_available.

And c_t units are sec^-1.

Let s be the distance covered assuming no wind.

V_inf = ds/dt

No assumptions about v_inf.

Combine everything, assume steady level flight and integrate and you get range for a turbojet (an equation on page 66

Again, assumptions are:

-no wind, steady, level flight

T parallel to V_inf

-Turbojet

What about props?

You end up with the equation on page 68

Solving the range integrals……

You get Range ofr a turbojet

R = 2/Ct etc etc (it's on the bottom of page 68

On page 69 is the method of finding Cl for Cl^1/2 / Cd max.. You stick Cl^1.2 over the two-praameter drag polar, take the derivative w/r/t Cl, then set the numerator =0 and solve for Cl.

Then, in order to find Cl^1/2 / Cd, you plug it back in/

Max range for a propeller

You get the Breguet range equation, on page 70.

And on the same page is a velocity equation to go along with that.

Endurance – the last topic for steady level flight

-How long can airplanes fly?

We are interested in t.

The two integrals for turbojet and propeller are on page 71.

Assumptions: steady, level flight

-epsilon = 0

And wind is irrelevant in the endurance calculation

Turbojet – let's solve integrals!

Assume constant L/D

You get…..

E 1/c_t * CL/Cd * ln(wo/w1)

(also assuming constant ct)

Max endurance happens when you fly at max L/D

-when you carry lots of fuel

And when you have an efficient engine.

Propeller….

The equation is E = npr/c sqrt(2rho_infs) CL^3/2 / Cd (1/sqrtw1) – 1/sqrt(w0))

Equations are on page 72.

If I want to fly at max endurance for a prop

Fly at Cl^3/2 / Cd max

-minimize c

-maximize npr

-carry lots of fuel

-fly at Sea level

An equation for Vcl/c^3/2 / Cd max is on page 73.

V_inf for max endurance for prop corresponds to minimum power required for prop.

FINSIHED STEADY LEVEL FLIGHT DISCUSSION!!

NOW

Start Accelerated Flight discussion!Accelerated Flight

**Level turn

**Pullup maneauver

-takeoff

-landing

Starting with a level turn

Theta = 0

There is a diagram on page 74

And equations of motion

Find the turn radius R and turn rate w.

Turn radius R = v_inf^2/gtanphi

W = gtanphi / V-inf

We're going to define a factor called "lodad factor" n = L/W

Insteady level flight, load factor = 1.

Basically the number f "g"'s that you feel.

Rearrange …. L/W = 1/cosphi = n

The more you bank, the greater load factor you have

Now let's do the equations with load factor in mind. Start with some geometery:

Sin^2phi + cospi^2 = 1….etc.

You get R = v_inf^2 / gsqrt(n^2-1) and w = g*sqrt(n^2 – 1))/ v_inf

These are on page 75

Sometimes we want to minimize R and mazimize w

As v_inf lowers, phi is the highest bank angle, bit load factor increases.

For a given V_inf, we can calculate min value of R and corresponding max value of w.

This will occur at max value of n.

There are 3 limitations on load factor:

-structural limit: n_max, str

-stall limit

And Thrust limit

N_max, str is independent of V_inf. We can find in Ra and max w by plugging in n_max,str into the equation

-Stall

(cl_max on wing)

L = W/cosphi = nW = 1/2rho_inf*v_62 s Cl

So, n_max, stall = 1/2W * rho_inf * v_inf^2 s Cl_max.

This IS a function of V_inf.

T_A -> insufficient thrust means we can't hold the turn

You get a really ugly equation for n that's just n_max thrust -> Ta, max.

_> all the n's can be plugged into R and w equations, getting Rmax, struct / Rmaxthrust, etc etc

Comparison plot of all of these is called a V-n diagram.

It's a plot of v_inf vs. n_max for structure, load constrant of Thrust Available and stall load factor.

Structure is independent of V_inf, so it's a straight line.****

Pull up maneauver

We have diagrams and EOM on page 81

R = V_inf^2 / g(n-1)

A comparison of R and w equations for pull up and level turn is on page 86!

This is the end of discussion re: instant pull up

This is JUST when you are starting a pull up and theta = 0.

N = load fact

Limits n passenger vs. structural.

NEXT TOPIC

Take-off!

Two segments:

Ground roll segment

-Airborne seegement

-to clear some obstacle of some height H_OB you have some runway length and an aircraft as it takes off.

S_g and s_a diagram on page 86

Key speeds associated with takeoff:

V_stall

-v_mcg (minimum control speed on ground) – where you're able to control the a/c aerodynamically

-vmca – minimum control speed in air

-v1 – decision speed (function of runway length)

-vr – rotation speed – one part comes off the ground

-vmu – minimum unstrick speed – enough lift so that tires aren't 100% in contact with the ground

-Vlo – lift-off speed – where you are totally airborne

These are have associations with regulations.

For example, v_lo = 1.1(Vstall)

A FBD of an airplane during ground roll segment is on page 87

Find There is a new drag polar during the ground roll segment

-actually in reality there's a thing called ground effect that makes Cl and alpha vary, but it's negligible for these purposes

-also you can end up having a "tail stike" OOPS.

Derivation of ground roll distance is on page 89.

If you assume some things are constant during GR, you get an equation for sg (page 90)

T>>R

You made a LOT of assumptions in this case, but this simplified model helps understand what sg is a function of.

It's a function of W^2

And proportional to 1/rho^2

As altitude increases, s_g increases

Due to the fact that T effects and L effects

S_g decreases as Thrust, wing area, and Clmax increases.

Airborne segment of take off

If you have alpha as a function of time, you can integrate EOM and determine the flight path bthat way.

But that's complicated!

For this class, let's assume constant pull-up, constant Radius ®

(diagram on page 90)

Derivation and equation on page 91.

NEXT – LANDING

You have

-Approach

-Flare

-Ground Roll

-most pilots slow dow just enough hto start taxiing, but calculations assume you're going to 0.

There are all these terms for portions of the landing

Va – approach speed

V-TD touchdown speed

Hf = flare height

R = flare radius

S_a – approach distance

‘sf – flare distance

Sfr – free roll distance

Sg = ground roll distance

Theta-a – approach angle

Theta-f flare angle

EOM and diagram is on page 92)

Landing

Flare – assume a “pull out” maneuver with constant R (or you can integrate EOM with varying angle of attack)

You use the same equations as pull up, pretty much)

-Ground rokll

EOM

T-D-R = mdv_inf . dt

And

L + N – W =0

-Exact same thing for landing but limits of integration switched

Dv_inf/dt should be less than zero b/c we are slowing down

The most important wuqtion after we land : how to slow down??

-Use brakes to increase mu_r

-Reduce lift (spoilers, parachutes)

-increase drag

0-apply reverse thrusters

-Arresting cables on aircraft carriers

Equation for ground roll is on page 94

THIS CONCLUDES FIXED-WING

Excepftfor some hybrid stuff

VTOL – vertical TO or landing

STOL – short takeoff or landing

-V/STOIL aircraft need to generate “lift” at low speeds

Lift is not aerodynamic lift in this case.

-capable of TO/landing

-hight T/W ratio

T-D-mu_r (W-L_ = mdV_inf/dt

Assume T>>D

Mu_r small, so T>>mu_r(W-L)

So….sg = W/gT * V0^2 / 2

Let V0 = 1.2Vstall

Then you have sg = 1/44 (W/S) all over g(T/W) * rho_inf * Cl_max

If you minimize W/S, T/W, weight, etc. (look at ppt)

Key V/STOL considerations

-balance

-Thrust and CG

-Control

-Yaw,pitch,roll

-hower, transition cruise

-Human factors

-pilot workload ,orientation, noise

-Environmental

-hot gas ingestion, noise footprint

V/STOL wheel of (mis)fortune (ppt)

-This show all VSTOL tested until 1997

It's VERY HARD to do vertical flight.

ROTORCRAFT

In vertical flight, the options are:

Climb, descend, and hover.

-There are 4 states in arotor:

(when you are a helicopter, you only want to be in one of these states at a time)

1) Normal working state
2) -hover or climbing
3) -power to rotor is coming from engine
4) States 2, 3, and 4 are UNDESIRABLE.
5) 2) Vorte Ring State
6) -slowly descending
7) -characterized by a lot of vibrations and turbulence
8) -power to rotor from engine
9) 3) turbulent wake state
10) -descending but faster than state 2
11) -more consistent by turbulent air (less crazy)
12) -extracting power from the air
13) -autorotation occurs
14) 4) windmill brake state
15) -rapid descent
16) -wind turbine operates at this state
17) -power is extracted from air

1) You can do a tiny bit of descent
2) 2) moving into the air you just pushed below you
3) 3) moving down as fast as the air you're pushing below
4) -No longer pushing air below you, falling and pushing rotor.

Momentum theory.

We use momentum to analyze thrust.

Considers the rotor as an actuator disk

Not thinking about the blades.gqw

This is also called “actuator disk theory”

-It’s a good representation at a distance from the rotor

-> FLOW ASSUMPTIONS

-incompressible, inviscid, steady, irrotatioal.

-1-D & uniform through disk & in far wak

-No swirl in the wake

This is a good place to start, but it obviously doesn’t hold.

We’re gonna think of a single-rotor helicopter.

You have a control volume

(diagram on page 103.

We have inflow and outflow:

-inflow is the top: rhoVs

Side: m-dot

-Outflow

Bottom: rhoV(s-A4) + rho(V+v4)A4

We can sat that m-dot = rhov4Af

Whatever is coming in through the sides is equal to the extra air moving at A4 zone at v4 velocity.

Additionally…the mass flow rate through the actuator disk is constant, meaning that v2 is equal to v3. We will call these v (a lower-case v.

Therefore: actuator disk is increasing the pressure across the rotor disk.

There is NO change in induced velocity

V = induced velocity at disk

LET'S TALK ABOUT MOMENTUM FLOW:

m-dotv

-infyou basically get from a lot of derivation that T = Arho(W + v4/2)v4

We get that the induced velocity is equal to v4/2. Induced velocity from actuator disk is half of the added velocity of the wake.

(a diagram of pressure and velocity across an actuator disk is on page 106.

Excess velocity in far wake is 2 times the velocity induced at the rotor.

Therefore the stream tube contracts.

Induced velocity = -V/2 * sqrt (V/2)^2 + T/2rhoA)

This gives us two solutions.

The negative soltuuion is where you are a wind turbine – energy is extracted from the flow. The positive is where you are a prop or rotor – energy is added to the flow.

(equation on page 106)

In HOVER

V = 0.

So your induced velocity in HOVER =>

V = sqrt T/2rhoA)

On to the power equation….

There are some rearrangements

P = T(V/@ + sqrt(V/@)^2 + T/2rhoA))

In hover, V = 0, so

P = T*sqrt(T/2rhoA)

In high altitude, Power increases.

If A disk is smaller Power increases.

Ideally, A disk is large.

Main takeaway:

-velocity at far wake is 2 times inducted velocity at disk

Actual power is gonna be higher – we made a LOT of assumptions!

Vertical flight

What forces are acting on a helicopter?

Thrust is up

Weight is down

In hover.

D = 0

We don't achieve ideal power…

There's this idea called "Figure of merit"

-ratio of ideal power for a rotor in hover (from momentum theory) to actual power obtained

-FM = Tv/Pactual

In hover, so FM = T(sqrt T/2rhoA) / P_actual.

This is not dimensionless

-opten hovercraft/rotorcraft are optimized rotor for HOVER. FM is around 0.8.

-Rotor w/ lower FM might be optimized for something else.

AN example of momentum theory is on page 109

Vertical flight continued

Non-dimensional forms:

C_t = thrust coeff = T/rhoA(OMEGAR)^2

Velocity at rotor tip

Power coeffifcient

Torque coefficient

In gover

P = OMEGA X Torque

Inflow induced (lambda_i) = v/ OMEGAR)

(All these non-dimensionals and equations are on page 109

Also we have something called disk loading

T/A.

The higher the disk loading, the higher induced velocity, the higher power.

Helicoptors T/A = 5-10 lb/ft^2

Tilt rotors T/A 20-40 lb/ft^2

VTOL T/A about 500 lb/ft^2

Vertical Flight

Talk about other states

Windmill brake state

-you're falling

-defined slipstream

-flow is up through the rotor

-V>2v

We can use momentum theory for this

It's just going the other direction.

(There's a graph of all the other states fon page 111.

Vortex Ring State

-descending

-no definite slipstream

-flow is recirculating and turbulent

-omentum theory is NOT applicable.

This is still powered flight

Power = T(V + v) > 0

At V + v = 0 , flow across rotor goes completely the other direction!

That's when you enter the turbulent Wake State

-Descending

-V+v < 0 but V+2v > 0

-momentum theory NOT applicable.

-Autorotation

T = T(V+v) < 0

Rotor powered by the air

-in practice, tou never want to descend that fast.

Time to talke about Forward flight!

(A diagram on page 112 and page 113)

With pWith a rotor, you have the advancing side and the retreating side. There's this reverse flow region.

-ChSpeed on air on advancing side is OMEGAR * V_inf

Speed of air on reteating side: OMEGAR – V_inf.

Challenges:

-you can get transonic flow on the forward blad

-you can get dynamic stall at leading edge of retreating side

-voortex being created by blades going into other blades

-differential Lift crated

-total lift decreases as Velocity increases

Therefore there is an upper limit on forward flight speed for helicoptors.

Let's tal about Pinduced = Tv

The important part is that

T = rhoAV2v for forward flight. (page 113)

From vertical flight, we get

T = rhoA(V+v)2v

Glauert's Hypothetsis says

No matter what, T = m-dot * 2v)/

m-dot varies depending on vrtical vs. forward flight.

m-dot = rhoA(V+v) for vertical flight

m-dot = rhoAV for forward flight, assuming V >>>> v

This gives good estimates but it's not based in any phsics or anything.

Thanks Glauert.

Some non-dimensionals

We have advance ratio:

Mu = velocity parallel to disk / tip speed. Which is ABOUT V_inf / OMEGA*R

Then we have lamda (inflow ratio)

Lamda = velocity across disk / tip speed) = V_infsinalpha + v / OMEGA*R

Lamda = mutanalpha + lamda_i

Mutanalpha is the fwd flight air coming in, and lamda_i is how the rotor is spinning, the induced portion of the inflow raio.

So…take everything with Thrust, lamda, C_t, mu, and make small angle assumetions and make it non-dim, you get…..

Approx. for high speed flight, mu > 0.2.

Mu is seldom above 0.4.

For forward flight:

C_T is about 2*lamdai*mu]

Lamda_i = C_T / 2mu

(derivation and equations on page 116 177

Performance

Ideal power = m-dot x deltaKE / unit mass

(usually it's deltaKE / unit time, but we have m-dot instead of m.

Ideal Power

P = T(Vsinalpha + v)P_actual = DV + Tv + Blade profile power (first term is parasite power, second term is induced power)

Parasite power is proportional to v_inf^3, so it ends up dominating Power.

When we make the small angle assumption for pwer, the induced power curve has an asymptotic solution close to zero (which is not realistic at all). Can only use small angle, and other asssumptions after a certain speed, V_inf.

Equations for coefficient of blade power are on page 119.

There are some corrections that can be added, with some assumotions about bhow many rotors you have and the shape of those rotors….

But anyway, the equation is all on page 119.

A SUMMARY OF MOMENTUM THEORY ASSUMPTIONS

-rotor is modeleld as actuator disk

-no detail about blades

-flow

-incompressible

-steady

-inviscid

-irrotational

-uniform through disk and in far wake

-1-D

-no swirl

Non-momentum theory approachges

-Blade Element Theory (BE Theory)

-divide the blad into a bunch of little sections and integrate over them

You can include a contrivution from momentum theory BEMT – Blade Elemenyt Momentum Contributition

(diagram on page 121)

From this , you get an “ideal twist” of the blade.

The angle which which the blade sees the oncoming air….which is a function of how far you are on the blad from the center.

Theta = c/R/r

Ideal twist!

Though ideal twist is hard to have at the hub.

Vortext theory

-Similar to Prandtly’s lifting line theory

-Kutta-Joukowski Theorem

-Biot-Savart Law

-wake representation

This relates blade lift (the K-J theorem)

To circulation

Delta-T = rho*OMEGAr)*GAMMA (rotation

BUT

CFD is replacing theory, b/c w/ a similar amount of effort, you can get better results often.

Takewaya = be able to understand assumptions about momentum theory and how to do it / use it.

Examples w/ performance analysis

(diagram on page 122)

Masx speed

-intersection of Pavail and Preq

At max speed, parasite power domincates

Parasite P = 1/2rhov^3f

V_max = (2/rhof (Pa – Pi – P0))^1/3

This is often iterative.

However, Power is not the limiting factor.

-most of the time: the limiting factor to speed is

-structural

-vibration

-etc.

Altitude effects:

-Pi small, so total P_req is down at higher alt

-However, P_actual decreases as you go faster

V_max decreases as altitude increases

PA = PA0 (rho/rho0)^n ; n = 0.7

(diagram in the notes) on page 123

Endurance of Rotorcraft

Endurange is hrs of loiter / lbs of fuel

You use forward speed where fuel consumption is minimum – not hover

Derivation oand equation on page 124.

See graph on page 124 as well.

SE (specific endurance) is 1/fuel consumption

Changes as weight changes, so we can't really easily integrate specific endurance to find total endurance….

We can do hour by hour, min by min.

Randge

dW_fuel burn / ds = dwf/dt / ds/dt is proportional to SFC P_rew / V

But we have a P/V term….sp… look at fig 1.

Specific range: (distance / fuel burn)

Like miles per gallon

We often fly at like 899 percent of best fuel consumption at best range.

Range equation is on page 125.

ROC for vertical Flight

Rate of climb is excess power / W

If in vertical flight

V = -V/2 +/- sqrt (V/2)^2 + T/srhoA)

V -> climb velocity

V -> induced velocity

Practically V << v_hover

So……

V is about v_hover – V/2

deltaPower = Piclimb – Phover.

Vc = 2deltaP / T

ROC – altitude effects

=P_hover increases with altitude

-Pavail decreases w altitude

-Delta P decreases with altitude and decresases with temperature.

(diagram on page 127

Absolute ceiling -> when P_R = P_max avail at V_inf = 0

Service Ceiling -> ROX_Max = 100 ft/min

Ground effect

-can have a big effect when taking off in high altitude

ROC For FORWARD FLIGHT

(diagram of motion is on page 128, with EOM also

Derivations and equations on page 128 and 129

Autorotation

-starts happening during turbulent wake state power is supplied to rotor by air

-RPM stabilizes

P_avil = 0

Rate of Descent = Preq/ W

As a general rule: descent rate is about 2 times hover induced velocity of that rotor.

Page 130 there is a diagram of Velocity vs P and the P_req curve